Coimisiún na Scrúduithe Stáit
State Examinations Commission

Scéimeanna Marcála	*Scrúduithe Ardteistiméireachta, 2005*
Matamaitic Fheidhmeach	*Ardleibhéal*

Marking Scheme	*Leaving Certificate Examination, 2005*
Applied Mathematics	*Higher Level*

M32

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General Guidelines

1 Penalties of three types are applied to candidates' work as follows:

Slips	- numerical slips	S(-1)
Blunders	- mathematical errors	B(-3)
Misreading	- if not serious	M(-1)

Serious blunder or omission or misreading which oversimplifies:
- award the attempt mark only.

Attempt marks are awarded as follows: 5 (att 2).

2 Mark all answers, including excess answers and repeated answers whether cancelled or not, and award the marks for the best answers.

3 Mark scripts in red unless candidate uses red. If a candidate uses red, mark the script in blue or black.

4 Number the grid on each script 1 to 10 in numerical order, not the order of answering.

5 Scrutinise **all** pages of the answer book.

6 The marking scheme shows one correct solution to each question. In many cases there are other equally valid methods.

1. **(a)** Car A and car B travel in the same direction along a horizontal straight road.
Each car is travelling at a uniform speed of 20 m/s.
Car A is at a distance of *d* metres in front of car B.
At a certain instant car A starts to brake with a constant retardation of 6 m/s^2.
0.5 s later car B starts to brake with a constant retardation of 3 m/s^2.

Find

(i) the distance travelled by car A before it comes to rest

(ii) the minimum value of *d* for car B not to collide with car A.

(i)

$$v^2 = u^2 + 2as$$

$$0 = 20^2 + 2(-6)s$$

$$s = \frac{100}{3} \quad \text{or} \quad 33.3$$ 5

(ii) Car B

$$s = ut + \tfrac{1}{2}at^2$$
$$s = 20(0.5) + 0$$
$$s = 10$$ 5

When brakes are applied:

$$v^2 = u^2 + 2as$$
$$0 = 20^2 + 2(-3)s$$
$$s = \frac{400}{6} \quad \text{or} \quad 66.6$$ 5

minimum value of $d = 66.6 + 10 - 33.3$ 5

$= 43.3$ 5

25

1 **(*b*)** A mass of 8 kg falls freely from rest. After 5 s the mass penetrates sand. The sand offers a constant resistance and brings the mass to rest in 0.01 s.

Find

(i) the constant resistance of the sand

(ii) the distance the mass penetrates into the sand.

(i) Speed after 5 s $v = u + at$

$v = 0 + 9.8(5)$

$v = 49$ 5

Sand $v = u + at$

$0 = 49 + a(0.01)$

$a = -4900$ 5

$mg - R = ma$ 5

$8(9.8) - R = 8(-4900)$

$R = 39278.4$ N 5

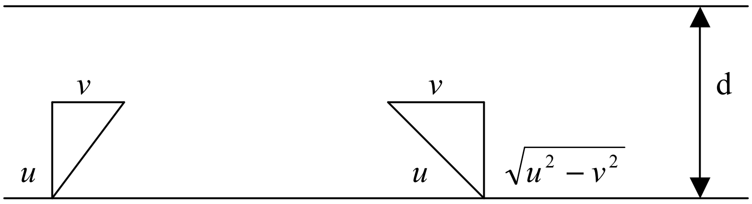
(ii) $v^2 = u^2 + 2as$

$0 = (49)^2 + 2(-4900)s$

$s = 0.245$ 5 25

2. **(a)** A woman can swim at u m/s in still water.
She swims across a river of width d metres. The river flows with a constant speed of v m/s parallel to the straight banks, where $v < u$.
Crossing the river in the shortest time takes the woman 10 seconds.

Find, in terms of u and v, the time it takes the woman to cross the river by the shortest path.



shortest time:

$$10 = \frac{d}{u} \quad \text{or} \quad d = 10u$$ 5

shortest path:

$$\text{speed} = \sqrt{u^2 - v^2}$$ 5

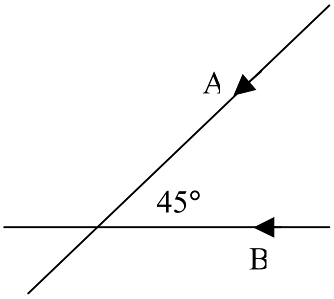
$$\text{time } = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{d}{\sqrt{u^2 - v^2}}$$ 5

$$= \frac{10u}{\sqrt{u^2 - v^2}}$$ 5

20

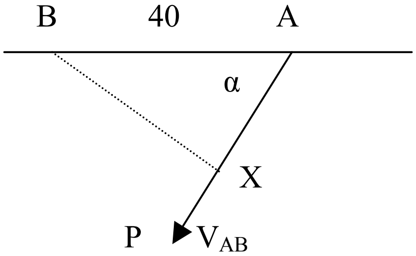
2 **(b)** Two straight roads intersect at an angle of 45°. Car A is moving towards the intersection with a uniform speed of p m/s. Car B is moving towards the intersection with a uniform speed of 8 m/s.
The velocity of car A relative to car B is $-2\vec{i}-10\vec{j}$.



At a certain instant car A is $220\sqrt{2}$ m from the intersection and car B is 136 m from the intersection.

(i) Find the value of p.

(ii) How far is car A from the intersection at the instant when the cars are nearest to each other?
Give your answer correct to the nearest metre.



(i) $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$ 5

$-2\vec{i}-10\vec{j}=\left(-\frac{p}{\sqrt{2}}\vec{i}-\frac{p}{\sqrt{2}}\vec{j}\right)-\left(-8\vec{i}\right)$ 5

$-2\vec{i}-10\vec{j}=\left(8-\frac{p}{\sqrt{2}}\right)\vec{i}-\frac{p}{\sqrt{2}}\vec{j}$

$\Rightarrow \quad p=10\sqrt{2}$ 5

(ii) A reaches the intersection in $\frac{220\sqrt{2}}{10\sqrt{2}}$ or 22 s

In this time B travels $8(22)$ or 176 m

and is now 40 m from the intersection

$\alpha = 90-\tan^{-1}\left(\frac{2}{10}\right)=78.69°$

$|\text{AX}| = 40\cos\alpha$

$= 40(0.1961)$ or 7.844 5

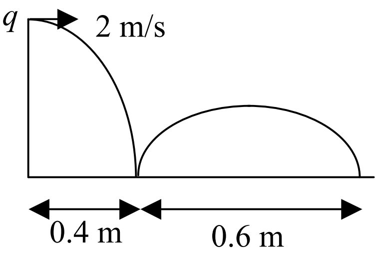
$\text{time}=\frac{|\text{AX}|}{\text{V}_{\text{AB}}} = \frac{7.844}{\sqrt{104}}$ or 0.769 5

In this time A travels $10\sqrt{2}(0.769)$ or 10.8 m

Ans 11 m. 5 30

3. **(a)** A ball is projected horizontally from a point *q* above a smooth horizontal plane with speed 2 m/s.
The ball first hits the plane at a point whose horizontal displacement from *q* is 0.4 m.
The ball next strikes the plane at a horizontal displacement of 1 m from *q*.
The coefficient of restitution between the ball and the plane is e.

Find the value of *e*.



First stage $\quad r\vec{i} = 0.4 \quad \Rightarrow 2(t) = 0.4$

$\Rightarrow \quad t = 0.2$ 5

$v\vec{j} = u + at$

$= 0 - 9.8(0.2)$

$= -1.96$ 5

Second stage $\quad r\vec{i} = 0.6 \quad \Rightarrow 2(t) = 0.6$

$\Rightarrow \quad t = 0.3$ 5

Rebound velocity $= 2\vec{i} + 1.96\, e\vec{j}$

$r\vec{j} = 0$

$1.96et - \frac{1}{2}gt^2 = 0$

$1.96e(0.3) - \frac{1}{2}g(0.3)^2 = 0$

$e = \frac{3}{4}$ 5

20

3 **(*b*)** A plane is inclined at an angle β to the horizontal. A particle is projected up the plane with initial velocity u at an angle α to the inclined plane.
The plane of projection is vertical and contains the line of greatest slope.

(*i*) Find the range of the particle on the inclined plane in terms of u, α and β.

(*ii*) Show that for a constant value of u the range is a maximum when

$$\alpha = 45° - \frac{\beta}{2}.$$

$$r_j = 0$$ 5

$$u\sin\alpha.t \;-\; \tfrac{1}{2}g\cos\beta.t^2 \;=\; 0$$

$$\Rightarrow \quad t \;=\; \frac{2u\sin\alpha}{g\cos\beta}$$ 5

$$\text{Range} \;=\; u\cos\alpha.t \;-\; \tfrac{1}{2}g\sin\beta.t^2$$ 5

$$= u\cos\alpha.\left(\frac{2u\sin\alpha}{g\cos\beta}\right) \;-\; \tfrac{1}{2}g\sin\beta..\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$ 5

$$= \frac{u^2}{g\cos\beta}\sin 2\alpha. \;-\; \frac{2u^2\sin\beta}{g\cos^2\beta}.\sin^2\alpha$$

$$\frac{dR}{d\alpha} \;=\; \frac{u^2}{g\cos\beta}2\cos 2\alpha. \;-\; \frac{2u^2\sin\beta}{g\cos^2\beta}.2\sin\alpha\cos\alpha$$ 5

$$= \frac{2u^2}{g\cos^2\beta}\{\cos 2\alpha.\cos\beta \;-\; \sin 2\alpha\sin\beta\}$$

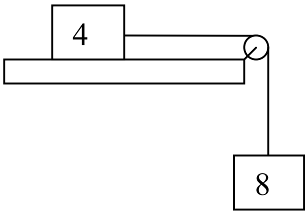
$$= \frac{2u^2}{g\cos^2\beta}\{\cos(2\alpha+\beta)\}$$

$$\frac{dR}{d\alpha} \;=\; 0 \quad\Rightarrow\quad \cos(2\alpha+\beta)=0$$

$$\Rightarrow \quad 2\alpha+\beta = 90°$$

$$\Rightarrow \quad \alpha = 45° - \frac{\beta}{2}$$ 5 30

4. **(a)** A particle of mass 4 kg rests on a rough horizontal table. It is connected by a light inextensible string which passes over a smooth, light, fixed pulley at the edge of the table to a particle of mass 8 kg which hangs freely under gravity.



The coefficient of friction between the 4 kg mass and the table is $\frac{1}{4}$.

The system starts from rest and the 8 kg mass moves vertically downwards.

Find

(i) the tension in the string

(ii) the force exerted by the string on the pulley.

(i)	$8g - T = 8f$	5
	$T - g = 4f$	5
	$8g - T = 2T - 2g$	5
	$10g = 3T$	
	$T = \frac{10g}{3}$	5
(ii)	$\text{Force} = \sqrt{T^2 + T^2}$	
	$= T\sqrt{2}$	
	$= \frac{10g\sqrt{2}}{3}$ or 46.2 N	5
		25

4 **(b)** Two particles of masses 3 kg and 5 kg are connected by a light inextensible string, of length 4 m, passing over a light smooth peg of **negligible radius**.
The 5 kg mass rests on a smooth horizontal table. The peg is 2.5 m directly above the 5 kg mass.
The 3 kg mass is held next to the peg and is allowed to fall vertically a distance 1.5 m before the string becomes taut.

(i) Show that when the string becomes taut the speed of each particle is

$$\frac{3\sqrt{3g}}{8} \text{ m/s.}$$

(ii) Show that the 3 kg mass will not reach the table.

(i)

$$v_1^{\,2} = u^2 + 2as$$
$$= 0 + 2(g)(1.5)$$
$$v_1 = \sqrt{3g}$$

5

$$3\left(\sqrt{3g}\right) = 3v + 5v$$

$$v = \frac{3\sqrt{3g}}{8}$$

5

(ii)

$$3g - T = 3a$$
$$T - 5g = 5a$$
$$a = -\frac{g}{4} \quad \text{or} \quad -2.45$$

5

3 kg mass

$$v^2 = u^2 + 2as$$
$$0 = \left(\frac{3\sqrt{3g}}{8}\right)^2 + 2\left(-\frac{g}{4}\right)s$$
$$s = \frac{27}{32} \quad \text{or} \quad 0.84$$

5

As $s < 1 \Rightarrow$ 3 kg mass will not reach the table

5

25

5. **(a)** Three identical smooth spheres P, Q and R, lie at rest on a smooth horizontal table with their centres in a straight line. Q is between P and R.
Sphere P is projected towards Q with speed 2 m/s. Sphere P collides directly with Q and then Q collides directly with R.

The coefficient of restitution for all of the collisions is $\frac{3}{4}$.

Show that P strikes Q a second time.

PQ PCM $\quad m(2)+m(0) = mv_1 + mv_2$

NEL $\quad v_1 - v_2 = -\frac{3}{4}(2-0)$

5

$$v_1 = \frac{1}{4}$$

$$v_2 = \frac{7}{4}$$

5

QR PCM $\quad m\left(\frac{7}{4}\right)+m(0) = mv_1 + mv_2$

NEL $\quad v_1 - v_2 = -\frac{3}{4}\left(\frac{7}{4}-0\right)$

5

$$v_1 = \frac{7}{32}$$

5

As $\frac{1}{4} > \frac{7}{32}$ P strikes Q a second time 5

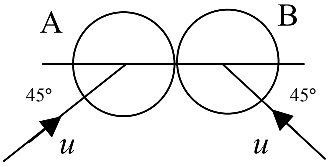
25

5 **(*b*)** A smooth sphere A, of mass m, moving with speed u, collides with an identical smooth sphere B moving with speed u.

The direction of motion of A, before impact, makes an angle $45°$ with the line of centres at impact.

The direction of motion of B, before impact, makes an angle $45°$ with the line of centres at impact.

The coefficient of restitution between the spheres is e.



(i) Find, in terms of e and u, the speed of each sphere after the collision.

(ii) If $e = \frac{1}{2}$, show that after the collision the angle between the directions of motion of the two spheres is $\tan^{-1}\left(\frac{4}{3}\right)$.

(i)

PCM $\quad m\frac{u}{\sqrt{2}} - m\frac{u}{\sqrt{2}} = mv_1 + mv_2$

NEL $\quad v_1 - v_2 = -e\left(\frac{u}{\sqrt{2}} + \frac{u}{\sqrt{2}}\right)$ 5

$$\Rightarrow \quad v_1 = -\frac{eu}{\sqrt{2}} \quad \text{and} \quad v_2 = \frac{eu}{\sqrt{2}}$$ 5

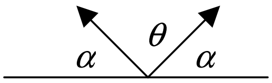
Speed of A $= \sqrt{\left(-\frac{eu}{\sqrt{2}}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2} = \frac{u}{\sqrt{2}}\sqrt{1+e^2}$

Speed of B $= \frac{u}{\sqrt{2}}\sqrt{1+e^2}$ 5

(ii)

$$e = \tfrac{1}{2} \quad \Rightarrow \quad v_1 = -\frac{u}{2\sqrt{2}} \text{ and } v_2 = \frac{u}{2\sqrt{2}}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2(2)}{1-4} = -\frac{4}{3}$$



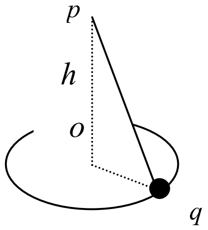
$$\theta = 180 - 2\alpha$$

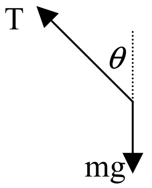
$$\tan\theta = \frac{\tan 180 - \tan 2\alpha}{1 + \tan 180 \tan 2\alpha}$$ 5

$$\tan\theta = \frac{0 - \left(-\frac{4}{3}\right)}{1+0} = \frac{4}{3}$$ 5

25

6. **(a)** A conical pendulum consists of a light inelastic string $[pq]$, fixed at the end p, with a particle attached to the other end q .
The particle moves uniformly in a horizontal circle whose centre o is vertically below p.
If $|po| = h$, find the period of the motion in terms of h.





$$T\cos\theta = mg$$ 5

$$T\sin\theta = mr\omega^2$$ 5

$$\tan\theta = \frac{mr\omega^2}{mg}$$ 5

$$\frac{r}{h} = \frac{mr\omega^2}{mg}$$

$$\frac{1}{\omega} = \sqrt{\frac{h}{g}}$$ 5

$$\text{Period} = \frac{2\pi}{\omega}$$

$$= 2\pi\sqrt{\frac{h}{g}}$$ 5

25

6 **(*b*)** A light elastic string of natural length a and elastic constant k is fixed at one end to a point o on a smooth horizontal table. A particle of mass m is attached to the other end of the string.
Initially the particle is held at rest on the table at a distance $2a$ from o, and is then released.

Show that the time taken for the particle to reach o is $\sqrt{\frac{m}{k}}\left(1+\frac{\pi}{2}\right)$.

Force $= -T$

$= -kx$

acceleration $= -\frac{k}{m}x$

SHM with $\omega = \sqrt{\frac{k}{m}}$ 5

Amplitude $= a$

Velocity $= \omega a = a\sqrt{\frac{k}{m}}$ 5

First stage time $= \frac{1}{4}T = \frac{\pi}{2}\sqrt{\frac{m}{k}}$ 5

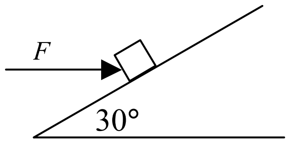
Second stage $s = ut + \frac{1}{2}at^2$

$a = a\sqrt{\frac{k}{m}}\ t + 0$

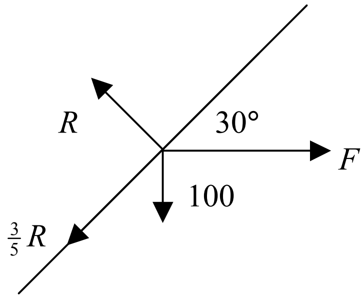
$t = \sqrt{\frac{m}{k}}$ 5

Total time $= \sqrt{\frac{m}{k}}\left(1+\frac{\pi}{2}\right)$ 5 25

7. **(a)** A particle of weight 100 N lies on a plane. The plane is inclined at 30° to the horizontal. A horizontal force F is applied to the particle. The coefficient of friction between the particle and the inclined plane is $\frac{3}{5}$.



Find the least value of F that will move the particle up the plane.



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$$R = 100\cos 30 + F\sin 30$$ 5

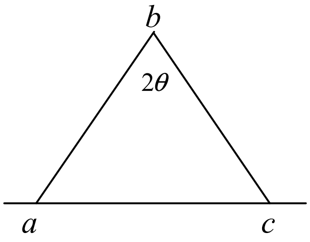
$$F\cos 30 = 100\sin 30 + \tfrac{3}{5}R$$ 5

$$F\cos 30 = 100\sin 30 + \tfrac{3}{5}\left(100\cos 30 + F\sin 30\right)$$ 5

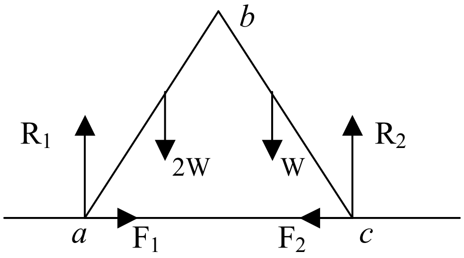
$$F = 180.1$$ 5

25

7 **(*b*)** Two uniform rods, [*ab*] and [*bc*], of equal length, are smoothly jointed at *b*.
They rest in a vertical plane with *a* and *c* on rough horizontal ground and $|\angle abc| = 2\theta$.
The weight of the rod [*ab*] is $2W$ and the weight of the rod [*bc*] is W.
The coefficient of friction at both *a* and *c* is μ.



Find the least value of μ, in terms of θ, necessary for equilibrium.



Vert $R_1 + R_2 = 3W$ 5

Take moments about *a* for system:

$$2W(1) + W(3) = R_2(4)$$
$$R_2 = \frac{5W}{4}$$
$$R_1 = \frac{7W}{4}$$

5

Take moments about *b* for *bc*:

$$F_2(\ell\cos\theta) + \mathrm{W}\left(\tfrac{1}{2}\ell\sin\theta\right) = R_2(\ell\sin\theta)$$
$$F_2 + \mathrm{W}\left(\tfrac{1}{2}\tan\theta\right) = \frac{5W}{4}(\tan\theta)$$
$$F_2 = \frac{3W}{4}(\tan\theta)$$

5

$$R_2 < R_1$$
$$\Rightarrow F_2 = \mu R_2$$

5

$$\frac{3W}{4}(\tan\theta) = \mu\frac{5W}{4}$$
$$\Rightarrow \mu = \frac{3}{5}\tan\theta$$

5

25

8. **(a)** Prove that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through its centre perpendicular to the rod is $\frac{1}{3}ml^2$.

Let M = mass per unit length

$$\text{mass of element} = M\{dx\}$$

$$\text{moment of inertia of the element} = M\{dx\}x^2 \qquad 5$$

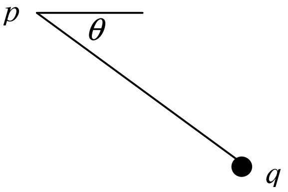
$$\text{moment of inertia of the rod} = M\int_{-\ell}^{\ell} x^2\,dx \qquad 5$$

$$= M\left[\frac{x^3}{3}\right]_{-\ell}^{\ell} \qquad 5$$

$$= \tfrac{2}{3}M\,\ell^3$$

$$= \tfrac{1}{3}m\,\ell^2 \qquad 5 \qquad 20$$

8 **(*b*)** A uniform rod [*pq*], of mass $9m$ and length $2l$, has a particle of mass $2m$ attached at q. The system is free to rotate about a smooth horizontal axis through p. The rod is held in a horizontal position and is then given an initial angular velocity $\sqrt{\frac{3g}{2l}}$ downwards.
The diagram shows the rod [*pq*] when it makes an angle θ with the horizontal.



(i) Show that when the rod makes an angle θ below its initial horizontal position, its angular velocity is

$$\sqrt{\frac{g(15+13\sin\theta)}{10l}}.$$

(ii) Hence, or otherwise, show that the rod performs complete revolutions about p.

(i)	moment of inertia $= \frac{4}{3}(9m)\ell^2 + (2m)(2\ell)^2$		
	$= 20m\ell^2$	5	
	Gain in KE = Loss in PE		
	$\frac{1}{2}I\omega_2^{\ 2} - \frac{1}{2}I\omega_1^{\ 2} = mgh$		
	$\frac{1}{2}(20m\ell^2)\omega_2^{\ 2} - \frac{1}{2}(20m\ell^2)\left(\frac{3g}{2\ell}\right) = 9mg(\ell\sin\theta)+2mg(2\ell\sin\theta)$	5,5	
	$\omega = \sqrt{\frac{g(15+13\sin\theta)}{10\ell}}$	5	
(ii)	$\theta = \frac{3\pi}{2}$		
	$\omega = \sqrt{\frac{g(15-13)}{10\ell}}$	5	
	> 0		
	⇒ rod performs complete revolutions	5	30

9. **(a)** An alloy is made of iron and aluminium.
A piece of the alloy has a mass of 0.441 kg and a volume of 75 cm^3.
The relative density of iron is 8 and the relative density of aluminium is 2.7.

Find

(i) the volume of iron in the piece of alloy

(ii) the mass of aluminium in the piece of alloy.

(i)

$$\text{Mass of iron} = 8000\,(V)\,10^{-6}$$ 5

$$\text{Mass of aluminium} = 2700\,(75 - V)\,10^{-6}$$ 5

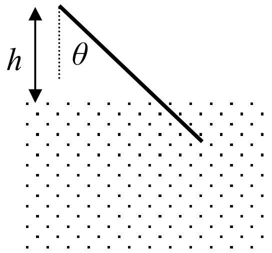
$$\begin{aligned} \text{Mass of alloy} &= \text{Mass of iron} + \text{Mass of alum.} \\ 0.441 &= 8000(V)10^{-6} + 2700(75 - V)10^{-6} \\ 441 &= 8\,(V) + 2.7\,(75 - V) \\ 238.5 &= 5.3V \end{aligned}$$ 5

$$V = 45 \text{ cm}^3$$ 5

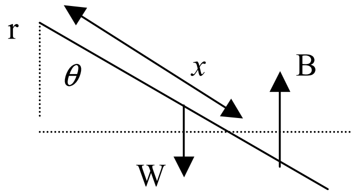
(ii)

$$\begin{aligned} \text{Mass of alluminium} &= 2700\,(75 - V)\,10^{-6} \\ &= 0.081 \text{ kg} \end{aligned}$$ 5 25

9 **(*b*)** A uniform rod of length ℓ and relative density s can turn smoothly about its upper end which is fixed at a height h above the surface of water. The rod is inclined at an angle θ to the vertical and is partially immersed in the water. The rod is at rest.



Show that $\cos\theta = \dfrac{h}{\ell\sqrt{1-s}}$, where $s < 1$.



$$\text{Buoyancy B} = \frac{W_I\, s_L}{s}$$

$$= \frac{\dfrac{(\ell - x)}{\ell}W\,(1)}{s} \quad \text{or} \quad \frac{W(\ell - x)}{\ell s}$$

5

Take moments about r :

$$B\left(\ell - \frac{(\ell - x)}{2}\right)\sin\theta = W\left(\frac{1}{2}\ell\right)\sin\theta$$

5,5

$$B\left(\frac{(\ell + x)}{2}\right) = W\left(\frac{1}{2}\ell\right)$$

$$\left(\frac{W(\ell - x)}{\ell s}\right)\left(\frac{(\ell + x)}{2}\right) = W\left(\frac{1}{2}\ell\right)$$

$$\ell^2 - x^2 = \ell^2 s$$

$$x^2 = \ell^2(1 - s)$$

$$x = \ell\sqrt{(1 - s)}$$

5

$$\cos\theta = \frac{h}{x}$$

$$= \frac{h}{\ell\sqrt{(1 - s)}}$$

5

25

10. **(a)** Solve the differential equation

$$x\frac{dy}{dx} - xy - y = 0$$

given that $y = 1$ when $x = 1$.

$$x\frac{dy}{dx} - xy - y = 0$$

$$\frac{dy}{dx} = \frac{y(x+1)}{x}$$

$$\int \frac{dy}{y} = \int \left(1 + \frac{1}{x}\right) dx$$ 5

$$\ln y = x + \ln x + C$$ 5

$$y = 1,\ x = 1 \Rightarrow \mathrm{C} = -1$$ 5

$$\mathrm{y} = e^{x+\ln x-1} \quad \text{or} \quad xe^{x-1}$$ 5

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10 **(*b*)** A mass of 9 kg is suspended at the lower end of a light vertical rope. Initially the mass is at rest. The mass is pulled up vertically with an initial pull on the rope of 137.2 N.
The pull diminishes uniformly at the rate of 1 N for each metre through which the mass is raised.

(i) Show that the resultant upward force on the mass when it is *x* metres above its initial position is

$$49 - x.$$

(ii) Find the speed of the mass when it has been raised 15 metres.

(iii) Find the work done by the pull on the rope when the mass has been raised by 15 m.

(i)

$$\text{Force} = 137.2 - x - 9g$$

$$= 49 - x$$ 5

(ii)

$$9v\frac{dv}{dx} = 49 - x$$ 5

$$9\int_0^v v\,dv = \int_0^{15}(49 - x)\,dx$$ 5

$$\left[\frac{9}{2}v^2\right]_0^v = \left[49x - \frac{x^2}{2}\right]_0^{15}$$ 5

$$\frac{9}{2}v^2 = 735 - 112.5$$

$$v = 11.76$$ 5

(iii)

$$\text{Work done} = \text{Change in K.E.}$$

$$= \tfrac{1}{2}(9)v^2 - 0$$

$$= 622.5$$ 5

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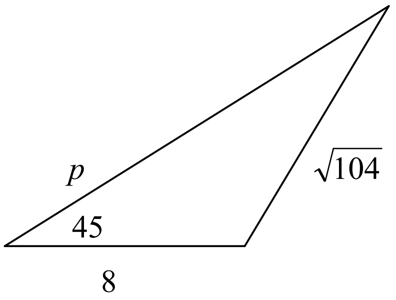
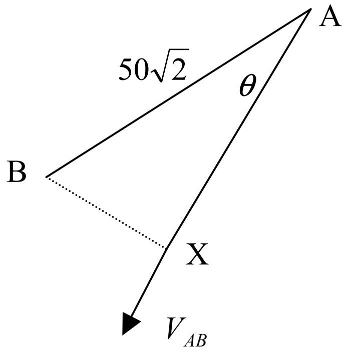
Some Alternative Solutions

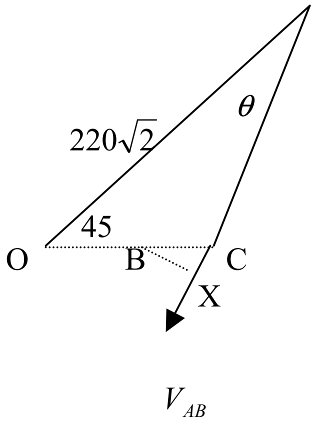
2(b)





(i)	$104 = p^2 + 64 - 2(8)p\left(\frac{1}{\sqrt{2}}\right)$	5	
	$p^2 - 8\sqrt{2}p - 40 = 0$	5	
	$\Rightarrow \quad p = 10\sqrt{2}$	5	
(ii)	B reaches the intersection in $\frac{136}{8}$ or 17 s In this time A travels $10\sqrt{2}(17)$ or $170\sqrt{2}$ m and is now $50\sqrt{2}$ m from the intersection		
	$\theta = 45° - \tan^{-1}\left(\frac{2}{10}\right) = 33.69°$		
	$\lvert AX \rvert = 50\sqrt{2}\cos 33.69$ $= 58.8349$	5	
	$\text{time} = \frac{\lvert AX \rvert}{V_{AB}} = \frac{58.8349}{\sqrt{104}}$ or 5.77	5	
	In this time A travels $10\sqrt{2}(5.77)$ or 81.60 m and is now $(81.60 - 50\sqrt{2})$m from the intersection		
	Ans 11 m.	5	30

2(b)



(ii) $$\theta = 45 - \tan^{-1}\left(\frac{2}{10}\right) = 33.69°$$

$$\frac{|\text{OC}|}{\sin 33.69} = \frac{220\sqrt{2}}{\sin 101.31}$$

$$|OC| = 176.0$$

$$|CB| = 40$$

$$|CX| = 40\cos 78.69 = 7.845$$

$$|AC| = \frac{220\sqrt{2}\sin 45}{\sin 101.31} = 224.357$$

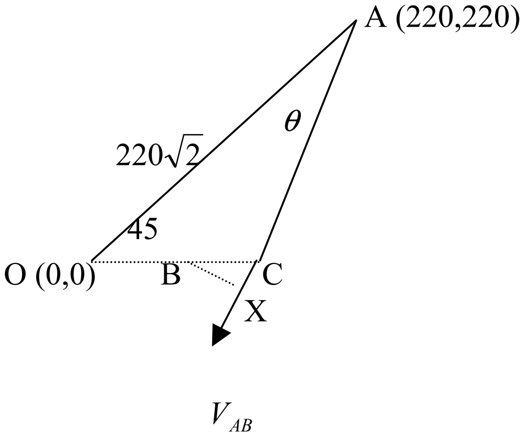
$$|AX| = |AC| + |CX| = 224.357 + 7.845 = 232.2$$ 5

$$\text{time} = \frac{|\text{AX}|}{\text{V}_{\text{AB}}} = \frac{232.2}{\sqrt{104}} \quad \text{or} \quad 22.769$$ 5

In this time A travels $10\sqrt{2}(22.769)$ or 322.0 m
and is now $322.0 - 220\sqrt{2} = 10.87$ m from intersection.

Ans 11 m. 5 15

2(b)



(ii) $$\theta = 45 - \tan^{-1}\left(\frac{2}{10}\right) = 33.69°$$

$$\frac{|\text{OC}|}{\sin 33.69} = \frac{220\sqrt{2}}{\sin 101.31}$$

$$|OC| = 176.0$$

Coords of C (176,0) and coords of B (136, 0)

Slope of AX $= \dfrac{220 - 0}{220 - 176} = 5$

equation of AX: $5x - y = 880$

equation of BX: $x + 5y = 136$

Coords of X $(174.46,\ -7.69)$

$$|\text{AX}| = \sqrt{(220 - 174.46)^2 + (220 + 7.69)^2}$$
$$= 232.2$$ 5

$$\text{time} = \frac{|\text{AX}|}{\text{V}_{\text{AB}}} = \frac{232.2}{\sqrt{104}} \quad \text{or} \quad 22.769$$ 5

In this time A travels $10\sqrt{2}(22.769)$ or 322.0 m
and is now $322.0 - 220\sqrt{2} = 10.87$ m from intersection.

Ans 11 m. 5 15

3(b)

$$r_j = 0$$ 5

$$u\sin\alpha.t \;-\; \tfrac{1}{2}g\cos\beta.t^2 \;=\; 0$$

$$\Rightarrow \quad t \;=\; \frac{2u\sin\alpha}{g\cos\beta}$$ 5

$$\text{Range} \;=\; u\cos\alpha.t \;-\; \tfrac{1}{2}g\sin\beta.t^2$$ 5

$$= u\cos\alpha.\left(\frac{2u\sin\alpha}{g\cos\beta}\right) \;-\; \tfrac{1}{2}g\sin\beta..\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$ 5

$$= \frac{2u^2\sin\alpha}{g}.\left(\frac{\cos\alpha}{\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos^2\beta}\right)$$

$$= \frac{2u^2\sin\alpha}{g}.\left(\frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos^2\beta}\right)$$

$$= \frac{2u^2\cos(\alpha+\beta)\sin\alpha}{g\cos^2\beta}$$

$$= \frac{u^2\{\sin(2\alpha+\beta) - \sin\beta\}}{g\cos^2\beta}$$ 5

This is a maximum when $\sin(2\alpha+\beta) = 1$

$$\Rightarrow \quad 2\alpha + \beta = 90°$$

$$\Rightarrow \quad \alpha = 45° - \frac{\beta}{2}$$ 5

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8 (b)

(ii) When the rod has rotated through 270°

Loss in KE = Gain in PE

$$\tfrac{1}{2}I\omega_1^{\,2} - \tfrac{1}{2}I\omega_2^{\,2} = mgh$$

$$\tfrac{1}{2}\left(20m\ell^2\right)\left(\frac{3g}{2\ell}\right) - \tfrac{1}{2}\left(20m\ell^2\right)\omega_2^{\,2} = 9mg(\ell) + 2mg(2\ell)$$

$$\omega = \sqrt{\frac{g}{5\ell}}$$ 5

> 0

$\Rightarrow$ rod performs complete revolutions 5

10